3 FATIGUE

A machine member loaded with a periodic stress that oscillates between some limits is subjected to stresses called *repeated, alternating,* or *fluctuating* stresses. Often the machine members fail under the action of repeated or fluctuating stresses. The most distinguishing characteristic of these failures is that the stresses have been repeated a very large number of times. Hence, the failure is called a *fatigue failure.* A fatigue failure begins with a small crack. Once the crack is initiated, the stress concentration effect becomes greater and the crack progress more rapidly. As the stressed area decreases in size, the stress increases in magnitude until, and the remaining area finally fails suddenly. A fatigue failure is characterized by two distinct regions. The first one is due to the progressive development of the crack, while the second one is due to the sudden fracture.

3.1 ENDURANCE LIMIT

Numerous tests have established that the ferrous materials have an *endurance limit* defined as the highest level of alternating stress that can be withstood indefinitely without failure. The symbol for endurance limit is S'_e. The endurance limit can be related to the tensile strength through the following relation

$$S'_e = \begin{cases} 0.504 S_{ut} & S_{ut} \leq 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \tag{3.1}$$

where S_{ut} is the minimum tensile strength. The prime mark on S'_e refers to the endurance limit of the test specimen itself, while the symbol S_e represent the endurance limit of a machine element subject to any kind of loading. Table 3.1 shows the endurance limit for various classes of cast irons.

The endurance limit is affected by some factors such that

$$S_e = k_a \, k_b \, k_c \, S'_e, \tag{3.2}$$

where k_a is the surface factor, k_b is the size factor (gradient factor), and k_c is the load factor. A rough guide for the values of these factors is given in Table 3.2.

3.1.1 Surface Factor k_a

The influence of the surface of the specimen is described by the modification factor k_a which depends upon the quality of the finishing. The following formula describe the surface factor

$$k_a = a S_{ut}^b, \tag{3.3}$$

where S_{ut} is the tensile strength and some values for a and b are listed in Table 3.3.

3.1.2 Size factor k_b

The size factor for bending and torsion can be expressed as follows

$$k_b = \begin{cases} \left(\dfrac{d}{0.3}\right)^{-0.1133} \text{in} & 0.11 \le d \le 2 \text{ in}, \\ \\ \left(\dfrac{d}{7.62}\right)^{-0.1133} \text{mm} & 2.79 \le d \le 51 \text{ mm}, \end{cases} \tag{3.4}$$

where d is the diameter of the bar. For larger sizes, the size factor varies from 0.06 to 0.075. There is no size effect for axial loading such that $k_b = 1$. To apply Eq. (3.4) for a nonrotating round bar in bending or for a noncircular cross section, we need to define the *effective dimension* d_e. This effective dimension is obtained by equating the volume of material stressed at and above 95 percent of the maximum stress to the same volume in the rotating beam specimen. When this two volumes are equated, the lengths cancel and only the areas have to be considered. For a rotating round section or a rotating hollow round, the 95 percent stress area is a ring having the outside diameter d and the inside diameter $0.95d$. Hence, the 95 percent stress area is

$$A_{0.95\sigma} = \frac{\pi}{4}[d^2 - (0.95d)^2] = 0.0766d^2. \tag{3.5}$$

For nonrotating solid or hollow rounds, the 95 percent stress area is twice the area outside of two parallel chords having a spacing of $0.95D$, where D is the diameter. Therefore the 95 percent stress area in this case is

$$A_{0.95\sigma} = 0.0105D^2. \tag{3.6}$$

Setting Eq. (3.5) equal to Eq. (3.6) and solving for d, we obtain the effective diameter

$$d_e = 0.370D, \tag{3.7}$$

which is the effective size of round corresponding to a nonrotating solid or hollow round.

A rectangular section of dimensions $h \times b$ (Fig. 3.1) has $A_{0.95\sigma} = 0.05hb$, and

$$d_e = 0.808(hb)^{1/2}. \tag{3.8}$$

For a channel section

$$A_{0.95\sigma} = \begin{cases} 0.5ab & \text{axis 1-1,} \\ 0.052xa + 0.1t_f(b-x) & \text{axis 2-2,} \end{cases} \tag{3.9}$$

where a, b, x, t_f are the dimensions of the channel section as depicted in Fig. 3.1(c). The 95 percent area for an I-beam section is (Fig. 3.1d)

$$A_{0.95\sigma} = \begin{cases} 0.10at_f & \text{axis 1-1,} \\ 0.05ba \quad t_f > 0.025a & \text{axis 2-2.} \end{cases} \tag{3.10}$$

3.1.3 Load factor k_c

The load factor has the following expression

$$k_c = \begin{cases} 0.923 & \text{axial loading} & S_{ut} \leq 220 \text{ kpsi (1520 MPa),} \\ 1 & \text{axial loading} & S_{ut} > 220 \text{ kpsi (1520 MPa),} \\ 1 & \text{bending,} & \\ 0.577 & \text{torsion and shear.} & \end{cases} \tag{3.11}$$

Hence, there is no size effect for specimens tested in axial or push-pull fatigue and there is a definite difference between the axial fatigue limit and that in reversed bending.

3.2 FLUCTUATING STRESSES

In design problems, it is frequently necessary to determine the stress of parts corresponding to the situation when the stress fluctuates without passing through zero (Fig. 3.2). A *fluctuating* stress is a combination of static plus completely reversed stress. The components of the stresses are depicted in Fig. 3.2(a), where σ_{min} is minimum stress, σ_{max} is the maximum stress, σ_a

is the stress amplitude or the alternating stress, σ_m is the midrange or the mean stress, σ_r is the stress range, and σ_s is the steady or static stress. The steady or static stress is not the same as the mean stress. It can have any value between σ_{min} and σ_{max}. This steady stress exists because of a fixed load and is usually independent of the varying portion of the load. The following relations between the stress components are useful

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}, \tag{3.12}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}. \tag{3.13}$$

The stress ratios

$$R = \frac{\sigma_{min}}{\sigma_{max}} \quad \text{and}$$

$$A = \frac{\sigma_a}{\sigma_m}, \tag{3.14}$$

are used to describe the fluctuating stresses.

3.3 CONSTANT LIFE FATIGUE DIAGRAM

Figure 3.3 illustrate the graphical representation of various combinations of mean and alternating stress. This diagram is called a *constant life fatigue diagram* because it has lines corresponding to a constant 10^6 cycle (or "infinite") life. The horizontal axis ($\sigma_a = 0$) corresponds to static loading. Yield and tensile strength are represented by points A and B, while the compressive yield strength is $-S_y$ (point A'). If $\sigma_m = 0$ and $\sigma_a = S_y$ (point A''), the stress fluctuates between $+S_y$ and $-S_y$. Line AA'' corresponds to fluctuations having a tensile peack of S_ym, and line $A'A''$ corresponds to compressive peaks equal to $-S_y$. Points C, D, E, and F correspond to $\sigma_m = 0$ for various values of fatigue life. Lines CB, DB, EB and FB are estimated lines of constant life. This empirical procedure to obtain constant life lines was developed by Goodman and these lines are called *Goodman lines*. The significance of the various areas on the diagram are

- Area A"$HCGA$ correspond to a life of at least 10^6 cycles and no yielding.
- Area $HCGA''H$ correspond to less than 10^6 cycles of life and no yielding.
- Area AGB in addition to area $A'HCGA$ correspond to 10^6 cycles of life when yielding is acceptable.

Example 3.1

Estimate the $S - N$ curve and a family of constant life fatigue curves pertaining to the axial loading of precision steel parts having $S_u = 120$ ksi, $S_y = 100$ ksi (Fig. 3.4). All cross-sections dimensions are under 2 in.

Solution

According with Table 3.2, the gradient factor $k_b = 0.9$. The 10^3 -cycle peak alternating strength for axially loaded material is $S = 0.75\, S_u = 0.75$ $(120) = 90$ ksi. The 10^6 -cycle peak alternating strength for axially loaded ductile material is $S_e = k_a k_b k_c S'_e$, where $S'_e = (0.5)(120) = 60$ ksi, $k_c = 1$, and from Fig. 3.5 $k_a = 0.9$. The endurance limit is $S_e = 48.6$ ksi. The estimated $S - N$ curve is plotted in Fig. 3.6. From the estimated $S - N$ curve results that the peak alternating strengths at 10^4 and 10^5 cycles are, respectively 76.2 and 62.4 ksi. The $\sigma_m - \sigma_a$ curves for $10^3, 10^4, 10^5$ and 10^6

cycles of life are given in Fig. 3.6.

3.4 FATIGUE LIFE FOR RANDOMLY VARYING LOADS

Predicting the life of parts stressed above the endurance limit is at best a rough procedure. For the large percentage of mechanical and structural parts subjected to randomly varying stress cycle intensity (for example, automotive suspension and aircraft structural components), the prediction of fatigue life is further complicated. The procedure for dealing with this situation is often called the *linear cumulative damage rule.* If a part is cyclically loaded at a stress level causing failure in 10^5 cycles, then each cycle of that loading consumes one part in 10^5 of the life of the part. If other stress cycles are interposed corresponding to a life of 10^4 cycles, each of these consumes one part in 10^4 of the life, and so on. When, on this basis, 100 percent of the life has been consumed, fatigue failure is predicted. The linear cumulative damage rule is expressed by the following equation

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + ... + \frac{n_k}{N_k} = 1 \quad \text{or} \quad \sum_{j=1}^{j=k} \frac{n_j}{N_j} = 1 \tag{3.15}$$

where $n_1, n_2 ..., n_k$ represent the number of cycles at specific overstress levels, and $N_1, N_2, ..., N_k$ represent the life (in cycles) at these overstress levels, as taken from the appropriate $S - N$ curve. Fatigue failure is predicted when the above sum is equal to 1.

Example 3.2

The stress fluctuation of a part during a 6 seconds of operation. The part has S_u = 500 MPa, and S_y = 400 MPa. The $S - N$ curve for bending is given in Fig. 3.7(c). Estimate the life of the part.

Solution

The 6-second test period includes, in order, two cycles of fluctuation (a), three cycles of fluctuation (b), and two cycles of (c). Each of these fluctuations corresponds to a point in Fig. 3.7(b). For a the stresses are $\sigma_m = 50$ MPa, $\sigma_a = 100$ MPa.

Points (a), (b), (c) in Fig. 3.7(b) are connected to the point $\sigma_m = S_u$, which gives a family of four "Goodman lines" corresponding to some constant life.

The Goodman lines intercept the vertical axis at points a' through c'. Points a through d correspond to the same fatigue lives as points a' through d'. These lives are determined from the $S - N$ curve in Fig.3.7(c). The life for a and a' can be considered infinite.

Adding the portions of life cycles b, c gives

$$\frac{n_b}{N_b} + \frac{n_c}{N_c} = \frac{3}{3 \times 10^6} + \frac{2}{2 \times 10^4} = 0.000011.$$

This means that the estimated life corresponds to 1/0.000011 or 90909 periods of 6-second duration. This is equivalent to 151.5 hours.

3.5 CRITERIA OF FAILURE

Various techniques are used to plot the fatique failure test results of a part subjected to fluctuating stress. For example, the *modified Goodman diagram* (Fig. 3.8) has the mean stress plotted on abscissa and all the stress components on the ordinate. The mean-stress line is a 45° line from the origin to the tensile strength of the part. The modified Goodman diagram consists from the lines constructed to S_e above and below the origin. The yield strength is also plotted on both axes, because yielding would be the criterion of failure if $\sigma_{max} > S_y$.

The fatigue diagram showing various criteria of failure is depicted is Fig. 3.9. This diagram is used for analysis and design purposes. The fatigue limit S_e or the finite-life strength S_f is plotted on the ordinate. The yield strength is plotted on the ordinate too. The mean-stress axis has the yield strength S_y and the tensile strength S_{ut} plotted along it. Four criteria of failure are depicted: the Soderberg, the modified Goodman, the Gerberg, and yielding. Only the Soderberg criterion guards against yielding. The linear criteria presented in Fig. 3.9 can be described by the equation of a straight line

$$\frac{x}{a} + \frac{y}{b} = 1. \tag{3.16}$$

In the above expresson a and b are the x and y intercepts, respectively. The equation for Soderberg line is

$$\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1. \tag{3.17}$$

Similarly, the modified Goodman relation is

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1. \tag{3.18}$$

The line representing the Gerber theory has a better chance of passing through the central portion of the failure points and should be a better predictor. This theory is also called the *Gerber parabolic relation* because the equation is

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_y}\right)^2 = 1. \tag{3.19}$$

The yielding line is described by the equation

$$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1. \tag{3.20}$$

The stresses σ_a and σ_m can replace S_a and S_m in Eqs. (3.17) to (3.19) if each strength is divided by the factor of safety n. Then the Soderberg equation becomes

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}. \tag{3.21}$$

The modified Goodman equation is

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}, \tag{3.22}$$

while the Gerber equation becomes

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1. \tag{3.23}$$

The meaning of these equations is illustrated in Fig. 3.10, using the modified Goodman theory as example. The safe-stress line through A is constructed parallel to the modified Goodman line. The safe-stress line is the locus of all sets of $\sigma_a - \sigma_m$ stresses having a factor of safety n, that is $S_m = n\sigma_m$ and $S_a = n\sigma_a$.

Figures Caption

Figure 3.1 Beams cross sections

a) solid round

b) rectangular section

c) channel section

d) web section

Figure 3.2 Time varying stresses

a) sinusoidal fluctuating stress

b) repeated stress

c) reversed sinusoidal stress

Figure 3.3 Constant life fatigue diagram

Figure 3.4 Axial loading cylinder

a) loading diagram

b) fluctuating load

Figure 3.5 Surface factor

Figure 3.6 Life diagram

Figure 3.7 Fatigue analysis of a cantilever beam

a) bending stress

b) stress fluctuation

c) life diagram

d) loading diagram

Figure 3.8 Goodman diagram

Figure 3.9 Various criteria of failure

Figure 3.10 Safe stress line

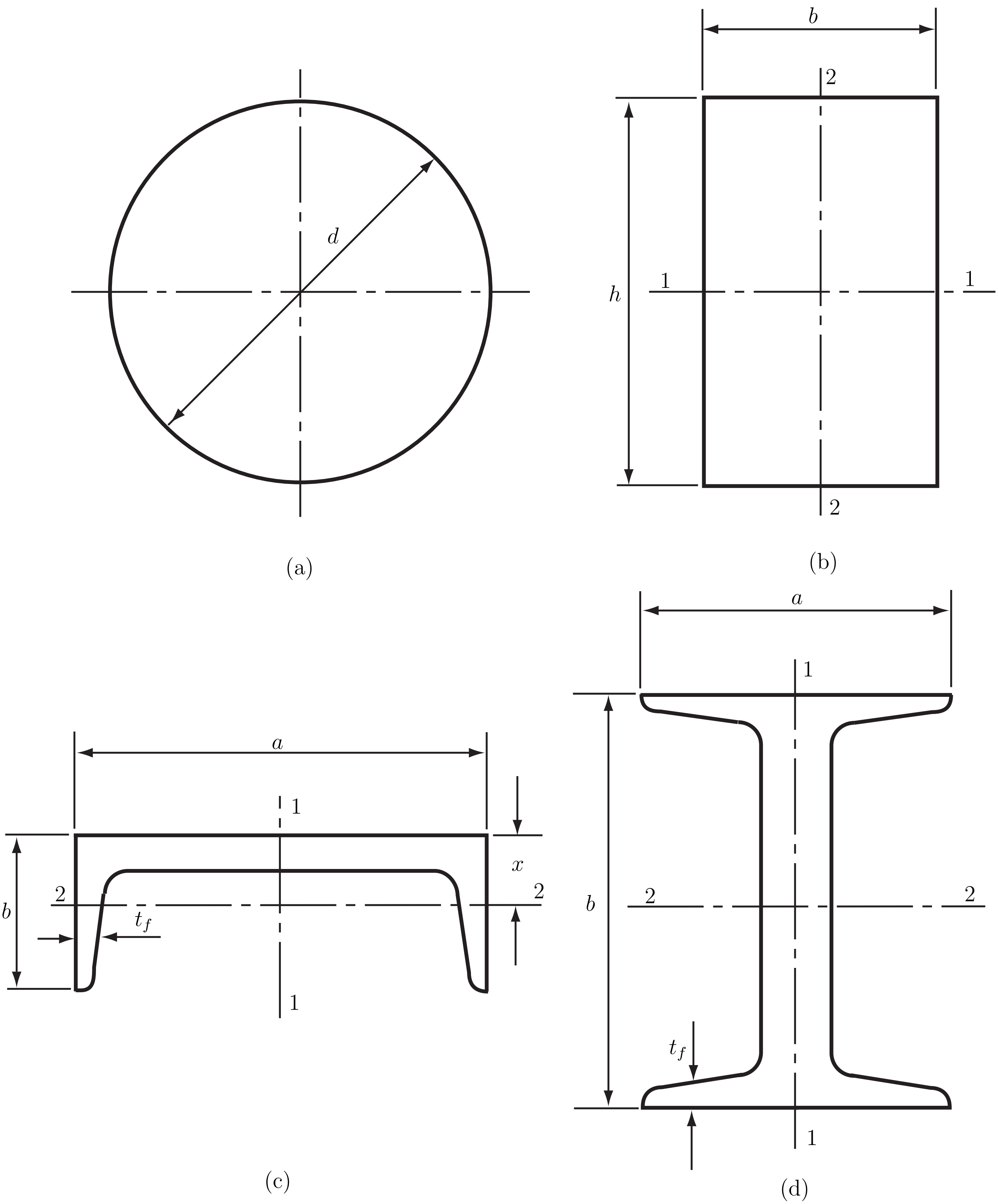


Figure 3.1

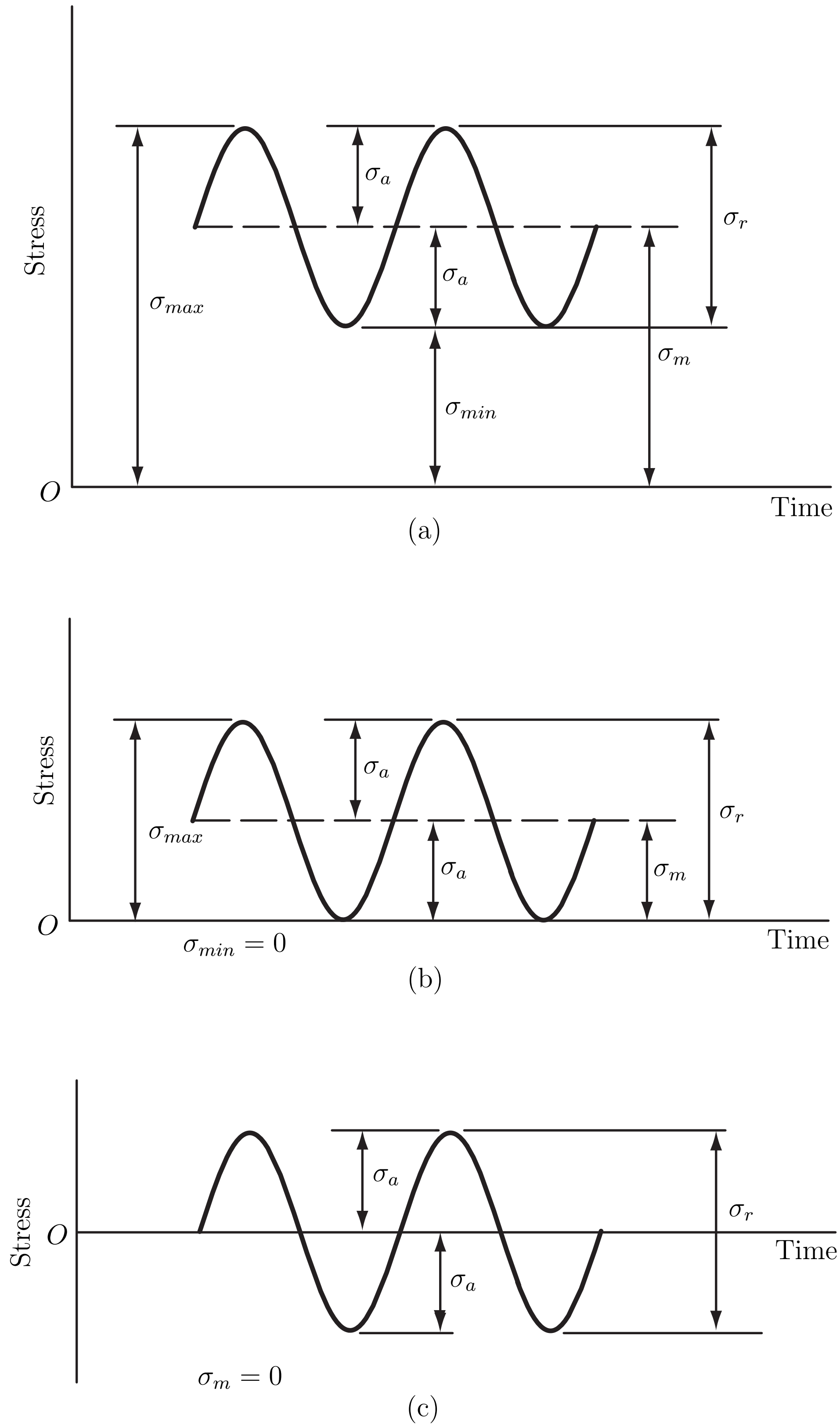
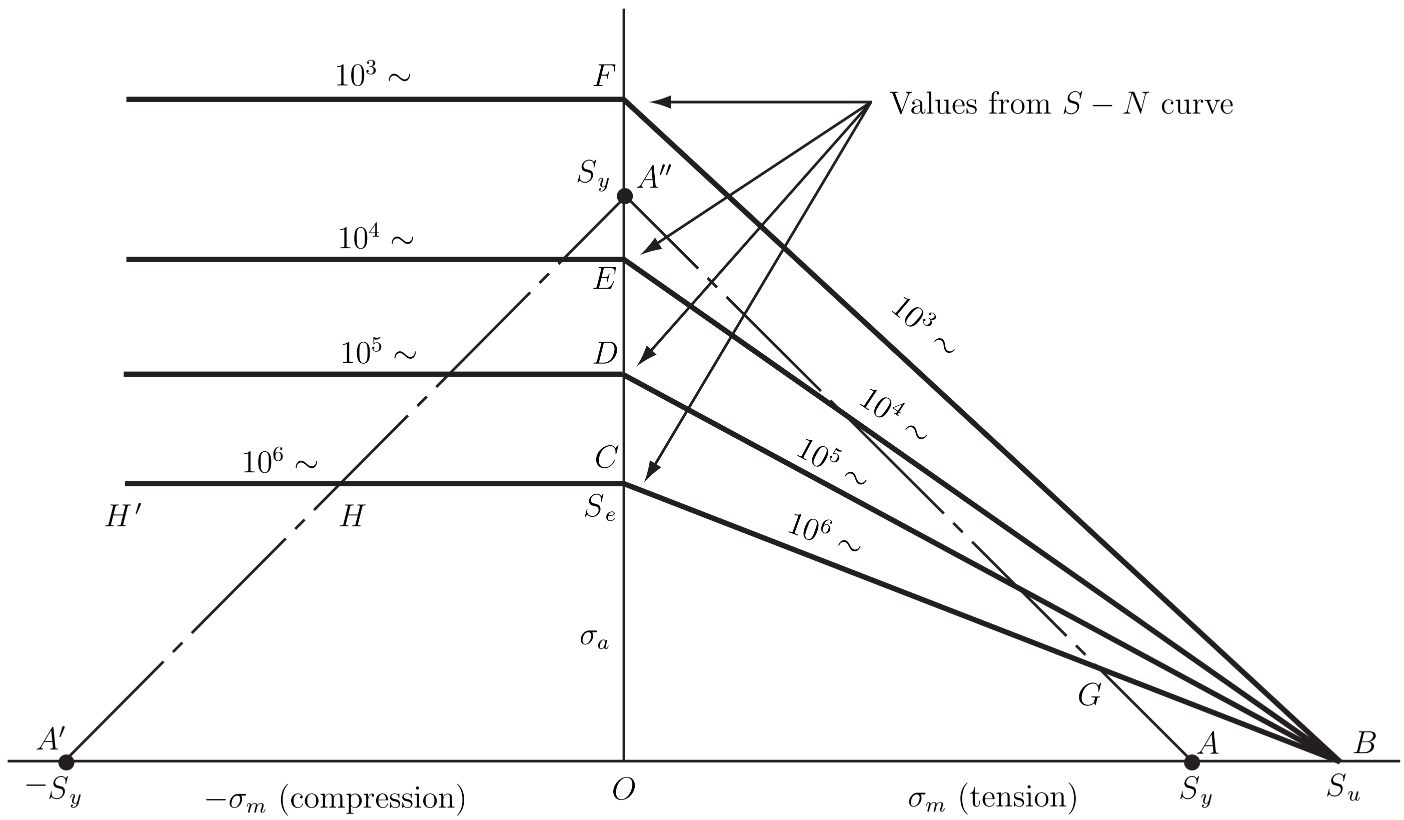
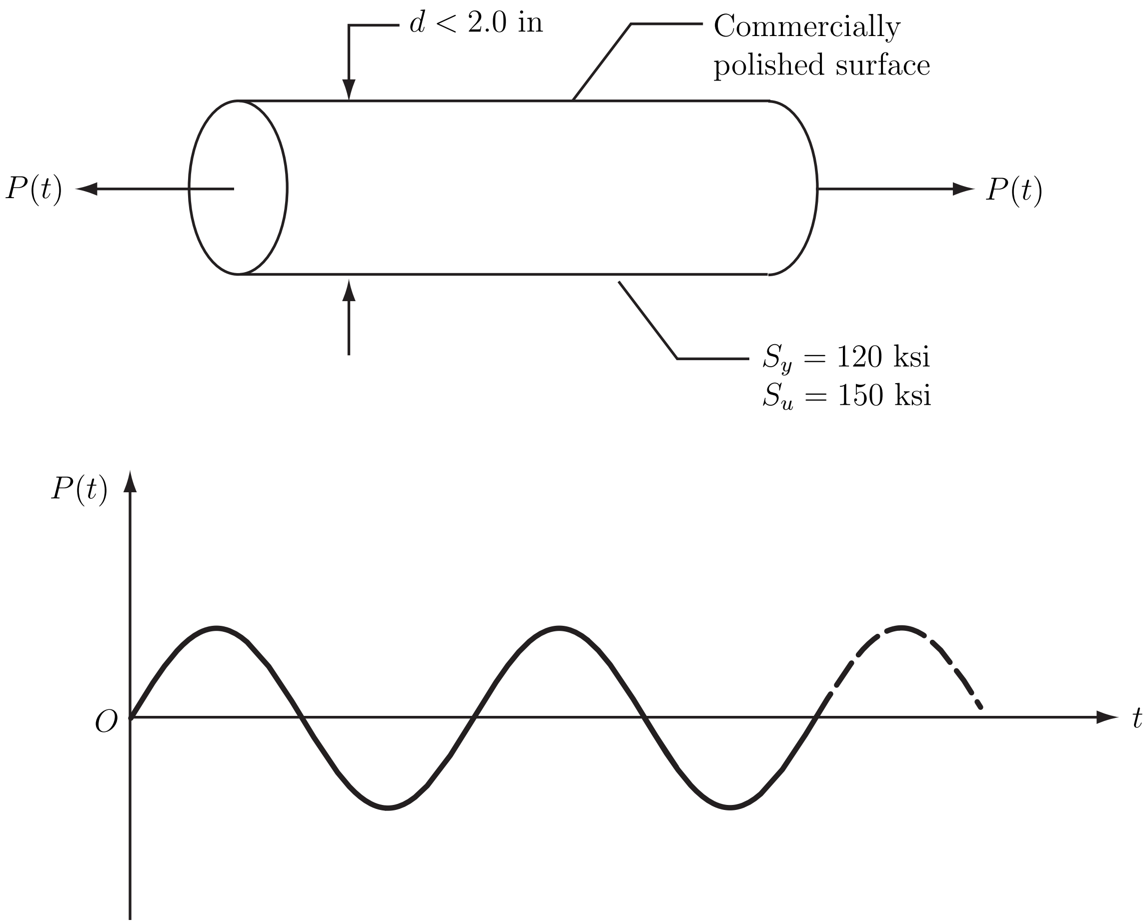

Stress
O
Time
σa
σa
σmax
σmin
σm
σr
(a)
Stress
O
Time
σa
σa
σmax
σm
σr
σmin = 0
(b)
Stress
O
Time
σa
σa
σr
σm = 0
(c)

Figure 3.2



10³ ~
F
Values from S − N curve
S_y
A″
10⁴ ~
E
10³ ~
10⁵ ~
D
10⁴ ~
10⁶ ~
C
10⁵ ~
H′
H
S_e
10⁶ ~
σ_a
G
A′
A
B
−S_y
−σ_m (compression)
O
σ_m (tension)
S_y
S_u

Figure 3.3



$d < 2.0$ in
Commercially polished surface
$P(t)$
$P(t)$
$S_y = 120$ ksi
$S_u = 150$ ksi
$P(t)$
O
t

Figure 3.4

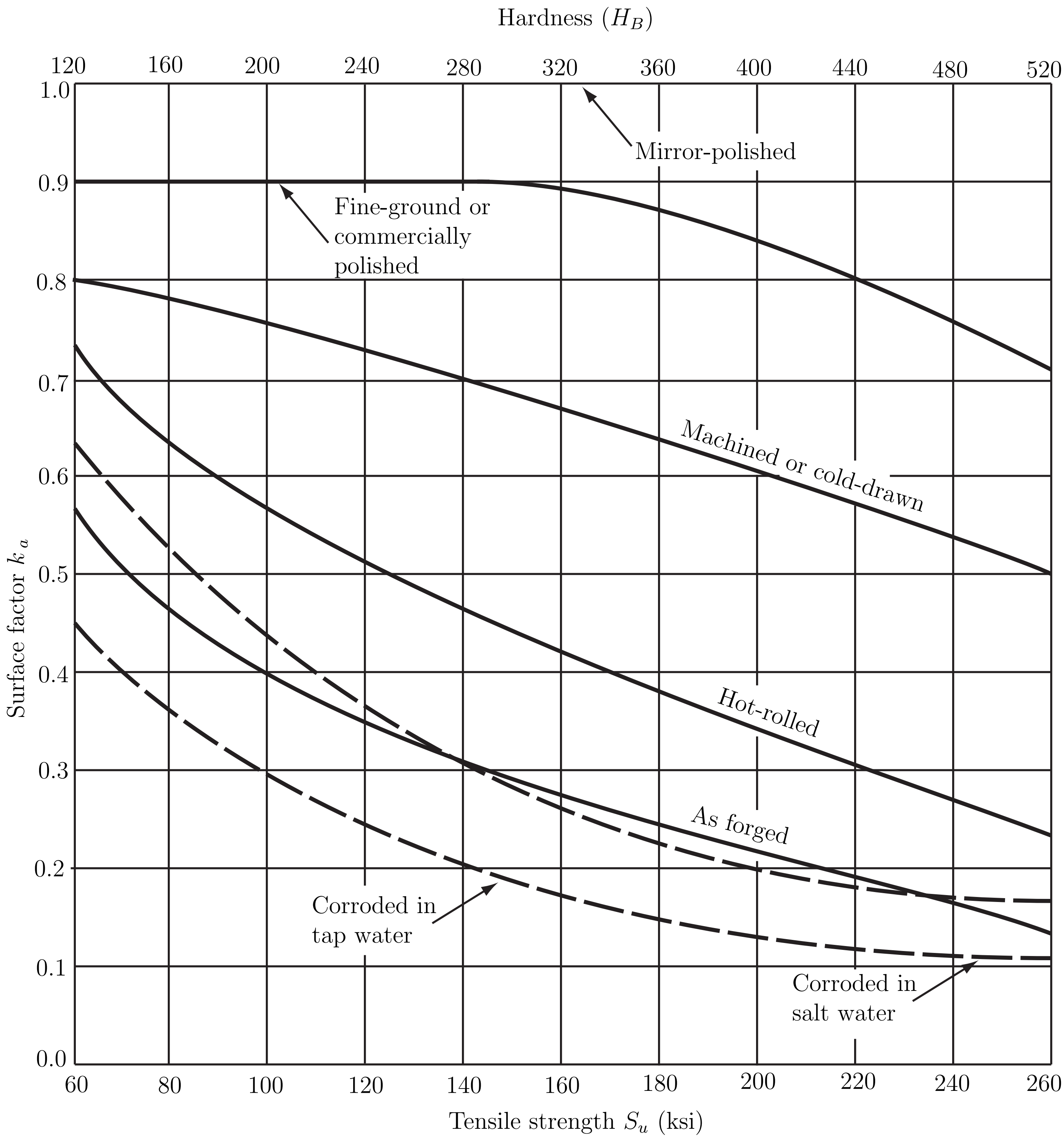
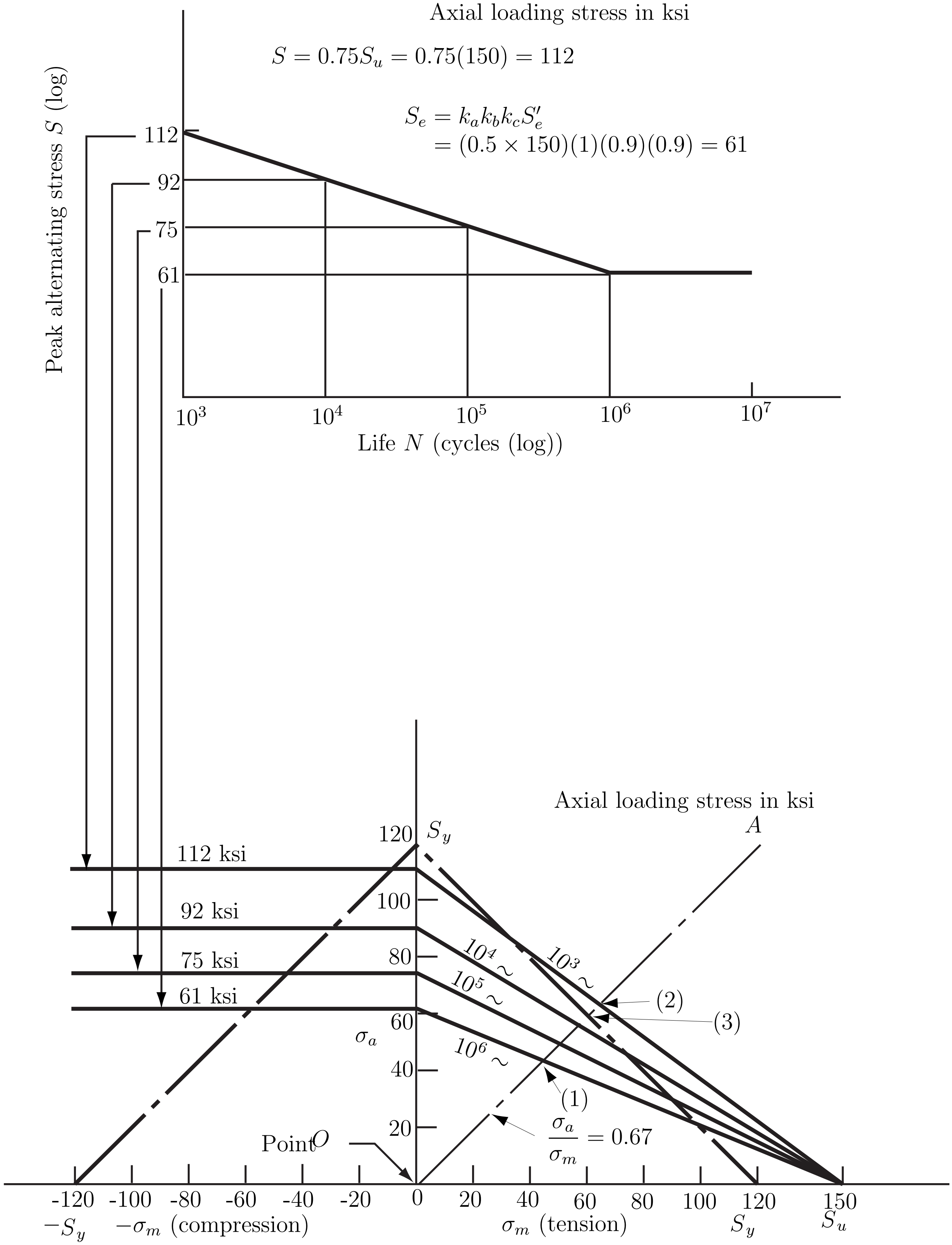
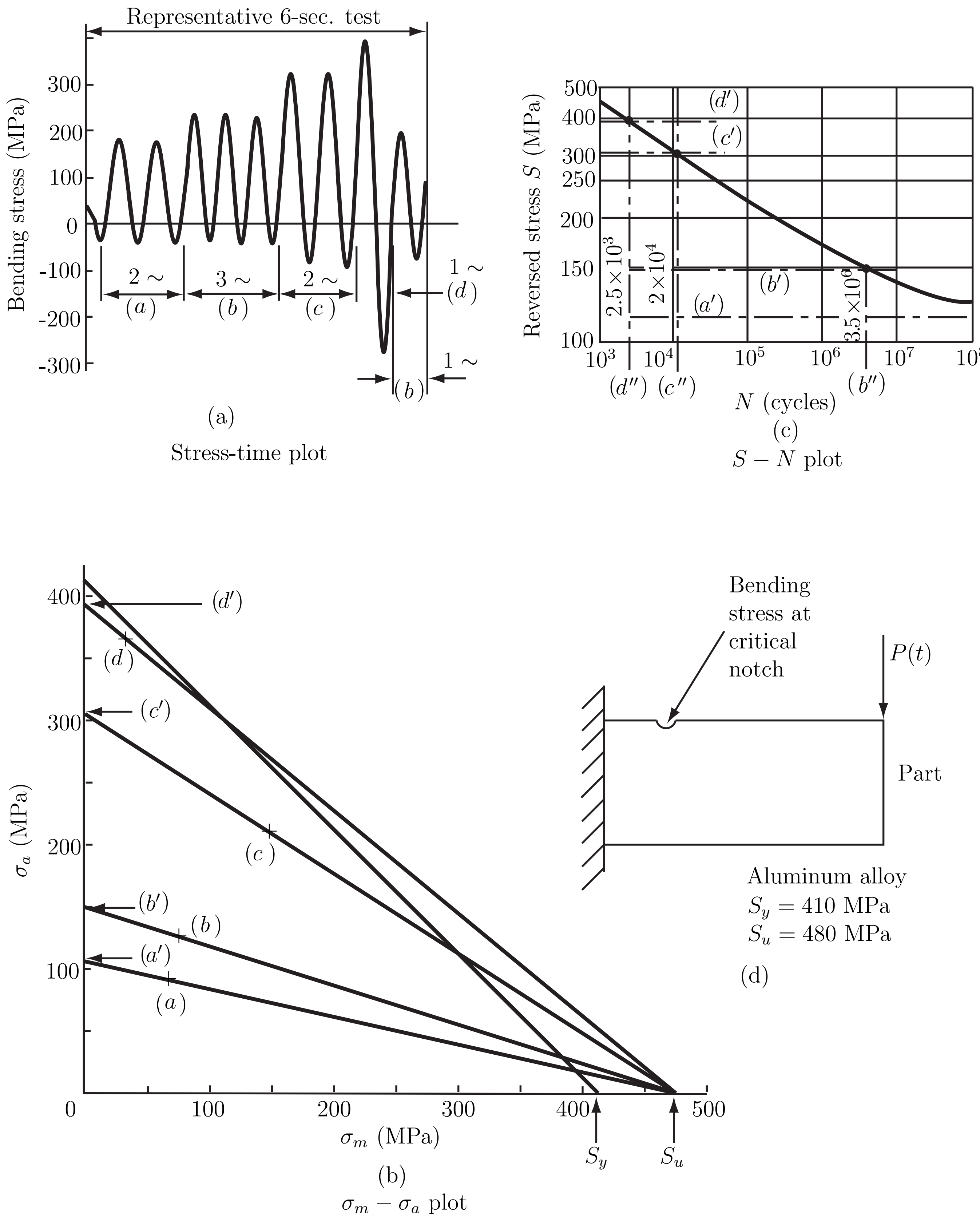

Hardness (H_B)
120
160
200
240
280
320
360
400
440
480
520
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Mirror-polished
Fine-ground or
commercially
polished
Machined or cold-drawn
Hot-rolled
As forged
Corroded in
tap water
Corroded in
salt water
Surface factor k_a
60
80
100
120
140
160
180
200
220
240
260
Tensile strength S_u (ksi)

Figure 3.5



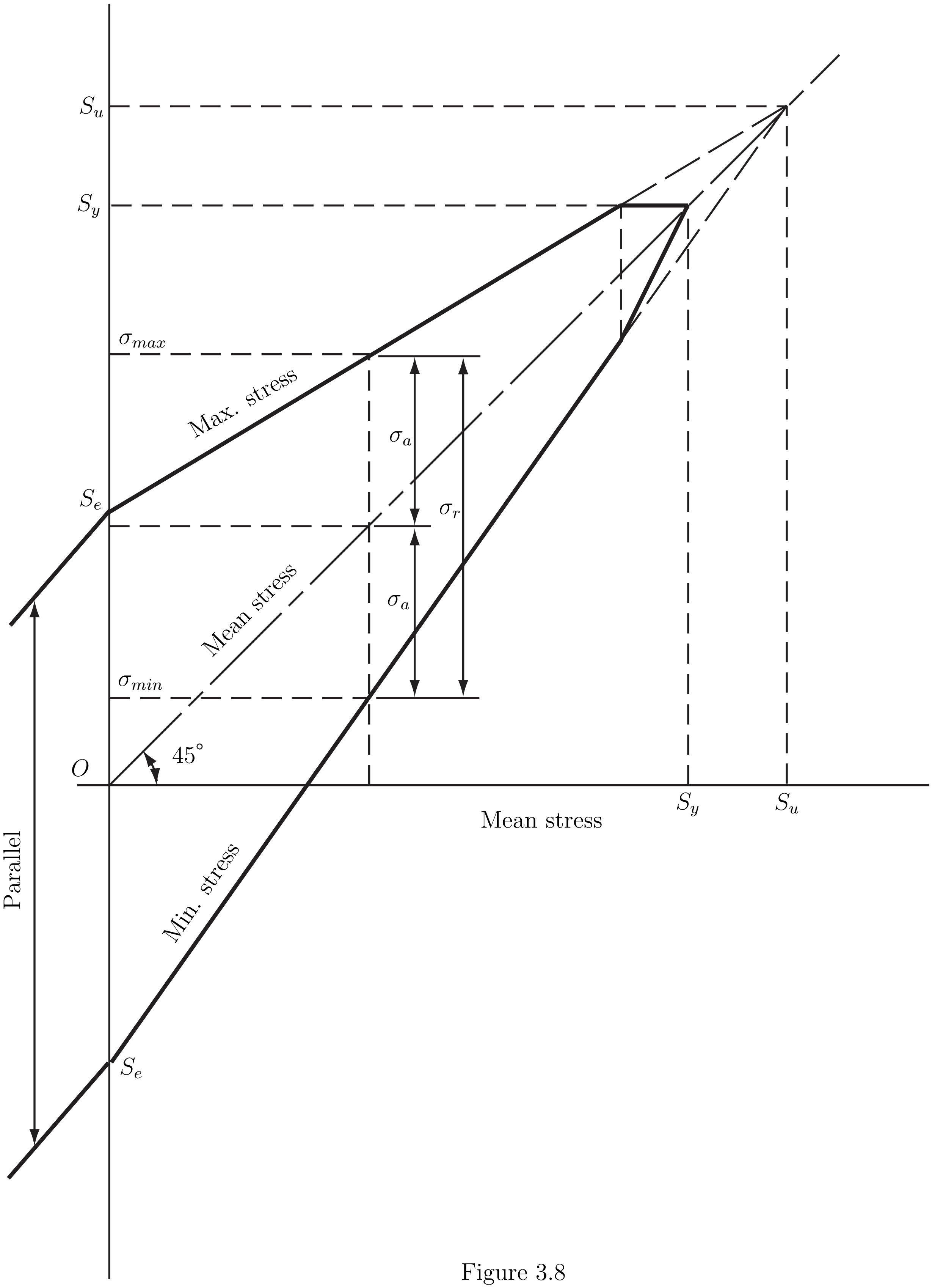
Axial loading stress in ksi
$S = 0.75S_u = 0.75(150) = 112$
$S_e = k_a k_b k_c S'_e$
$= (0.5 \times 150)(1)(0.9)(0.9) = 61$
Peak alternating stress S (log)
112
92
75
61
10^3
10^4
10^5
10^6
10^7
Life N (cycles (log))
Axial loading stress in ksi
A
120
S_y
112 ksi
92 ksi
75 ksi
61 ksi
100
80
60
40
20
σ_a
10^4
10^3
10^5
10^6
(1)
(2)
(3)
$\frac{\sigma_a}{\sigma_m} = 0.67$
Point O
-120
-100
-80
-60
-40
-20
0
20
40
60
80
100
120
150
$-S_y$
$-\sigma_m$ (compression)
σ_m (tension)
S_y
S_u

Figure 3.6



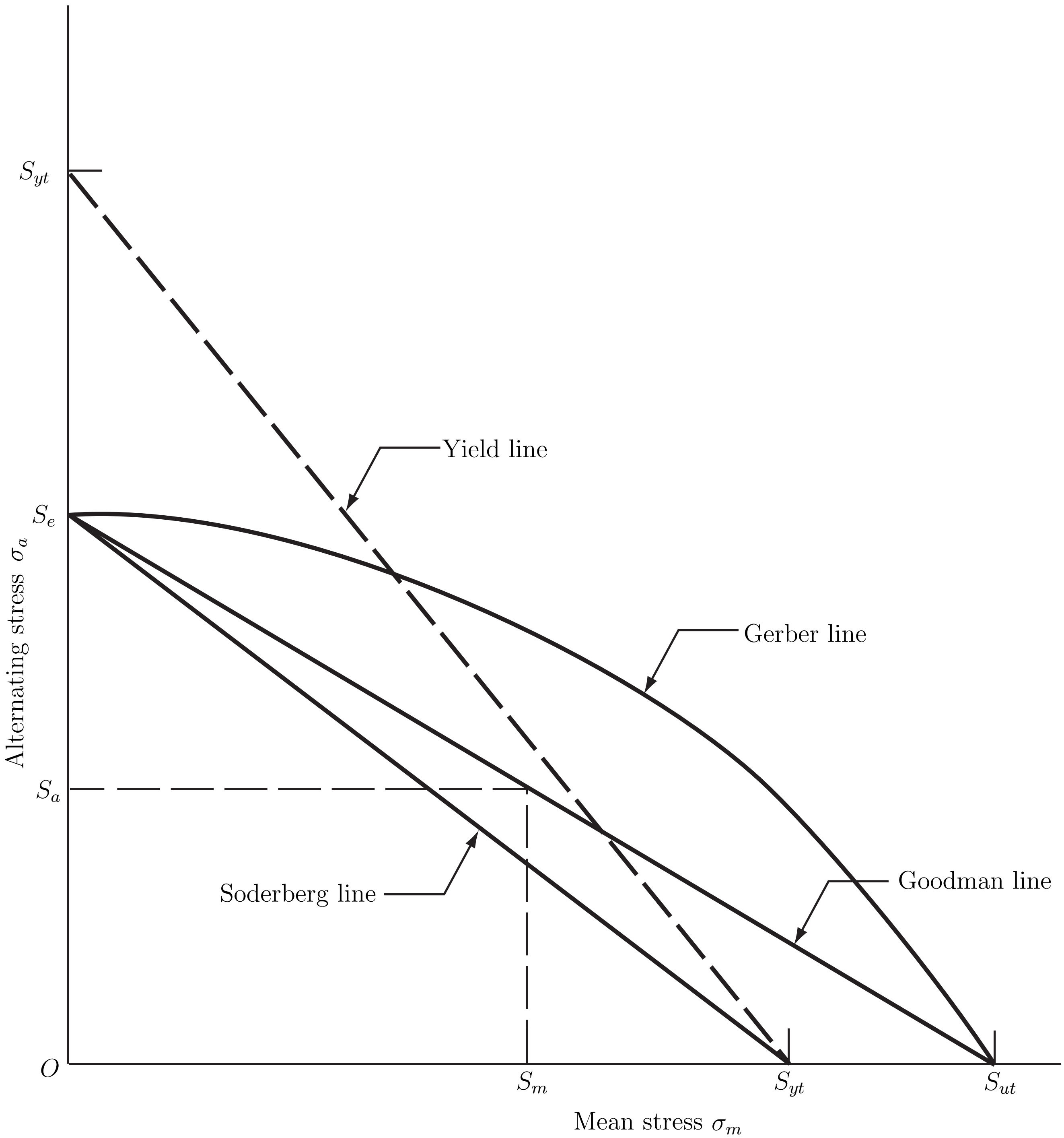
Representative 6-sec. test
Bending stress (MPa)
300
200
100
0
-100
-200
-300
2 ~
(a)
3 ~
(b)
2 ~
(c)
1 ~
(d)
1 ~
(b)
(a)
Stress-time plot
500
400
300
250
200
150
100
Reversed stress S (MPa)
(d′)
(c′)
(b′)
(a′)
2.5×10^3
2×10^4
3.5×10^6
10^3
10^4
10^5
10^6
10^7
10^8
(d″) (c″)
(b″)
N (cycles)
(c)
S − N plot
σa (MPa)
400
300
200
100
(d′)
(d)
(c′)
(c)
(b′)
(b)
(a′)
(a)
0
100
200
300
400
500
σm (MPa)
Sy
Su
(b)
σm − σa plot
Bending stress at critical notch
P(t)
Part
Aluminum alloy
Sy = 410 MPa
Su = 480 MPa
(d)

Figure 3.7



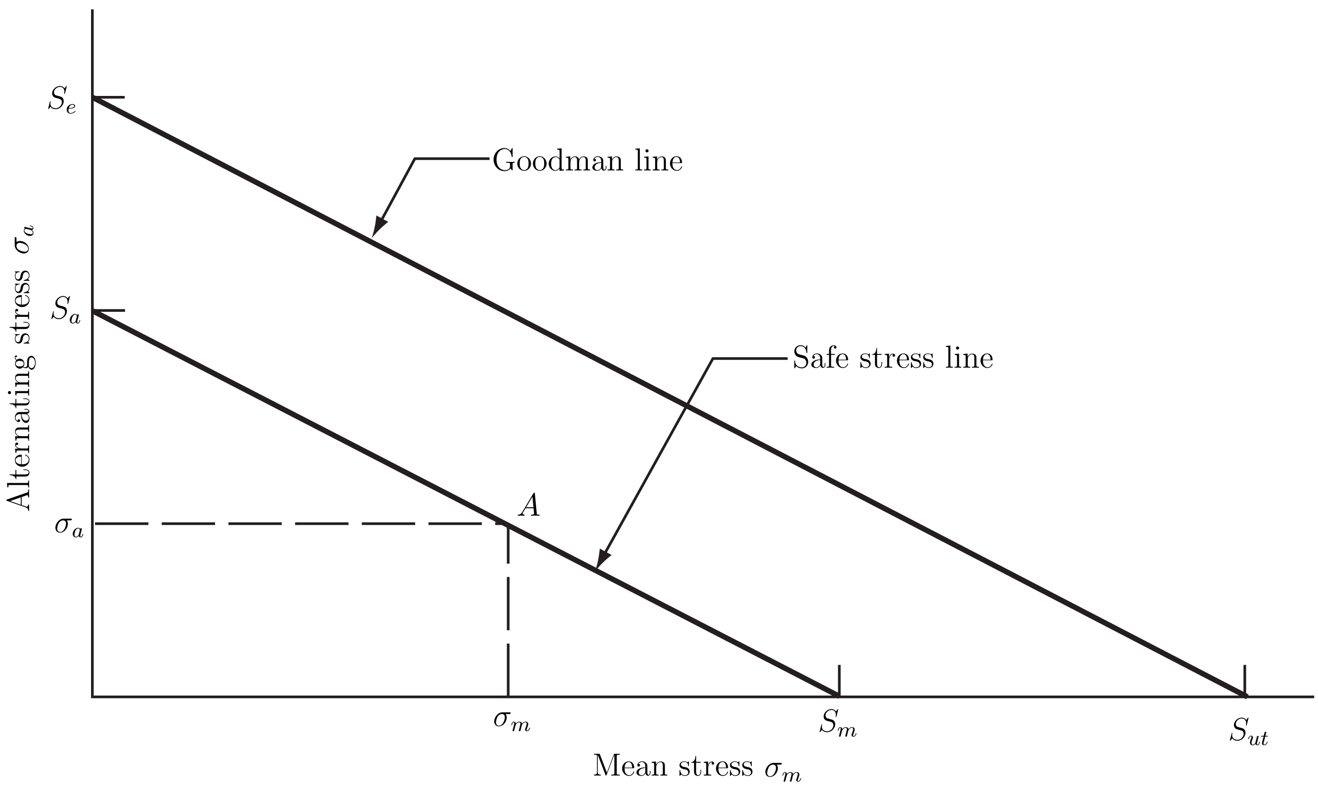
S_u
S_y
σ_{max}
Max. stress
σ_a
σ_r
S_e
Mean stress
σ_a
σ_{min}
45°
O
S_y
S_u
Mean stress
Parallel
Min. stress
S_e

Figure 3.8



S_{yt}
Yield line
S_e
Gerber line
Alternating stress σ_a
S_a
Goodman line
Soderberg line
O
S_m
S_{yt}
S_{ut}
Mean stress σ_m

Figure 3.9



S_e
Goodman line
S_a
Safe stress line
Alternating stress σ_a
σ_a
A
σ_m
S_m
S_{ut}
Mean stress σ_m

Figure 3.10

Table 3.1. Typical Properties of Gray Cast Iron

ASTM NUMBER	TENSILE STRENGTH S_{ut}, kpsi	COMPRESSIVE STRENGTH S_{uc}, kpsi	SHEAR MODULUS OF RUPTURE S_{su}, kpsi	MODULUS OF ELASTICITY, Mpsi		ENDURANCE LIMIT S_e, kpsi	BRINELL HARDNESS H_B	FATIGUE STRESS CONCENTRATION FACTOR K_f
				TENSION	TORSION			
20	22	83	26	9.6-14	3.9-5.6	10	156	1.00
25	26	97	32	11.5-14.8	4.6-6.0	11.5	174	1.05
30	31	109	40	13-16.4	5.2-6.6	14	201	1.10
35	36.5	124	48.5	14.5-17.2	5.8-6.9	16	212	1.15
40	42.5	140	57	16-20	6.4-7.8	18.5	235	1.25
50	52.5	164	73	18.8-22.8	7.2-8.0	21.5	262	1.35
50	62.5	187.5	88.5	20.4-23.5	7.8-8.5	24.5	302	1.50

Source: Joseph E. Shigley, Charles R. Mischke, *Mechanical Engineering Design*, 5d ed.,McGraw-Hill, 1989, p. 123.

Table 3-2. Generalized Fatigue Strength Factors for Ductile Materials

	BENDING	AXIAL	TORSION
a. Endurance limit			
$S_e = k_a k_b k_c S'_e$, where S'_e is the specimen endurance limit			
k_c (load factor)	1	1	0.58
k_b (gradient factor) diameter< (0.4 in or 10 mm) (0.4 in or 10 mm) < diameter < (2 in or 50 mm)	 1 0.9	 0.7 - 0.9 0.7 - 0.9	 1 0.9
k_a (surface factor)	See Fig. 3.5		
b. 10^3 -cycle strength	0.9 S_u	0.75 S_u	0.9 S_{us}^a

$^a S_{us} \approx 0.8 S_u$ for steel; $S_{us} \approx 0.7 S_u$ for other ductile materials.
Source: Robert C. Juvinall, Kurt M. Marshek, *Fundamentals of Machine Component Design*, 2nd ed.,John Wiley & Sons, 1991, p. 270.

Table 3-3. Surface Finish Factor

SURFACE FINISH	FACTOR a		EXPONENT b
	kpsi	MPa	
Ground	1.34	1.58	-0.085
Machined or cold-drawn	2.70	4.51	-0.256
Hot-rolled	14.4	57.7	-0.718
As forged	39.9	272.0	-0.995

Source: Joseph E. Shigley, Charles R. Mischke, *Mechanical Engineering Design*, 5d ed.,McGraw-Hill, 1989, p. 123.